COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

ELEN E3106/4106 Lecture 8

p-n Junctions Part I

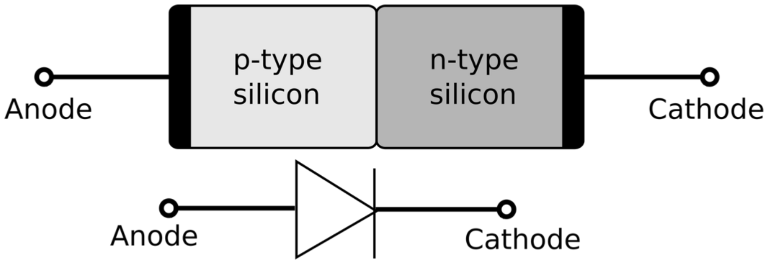
Outline

- Simple theory to p-n junctions
- Junctions in equilibrium
- Contact Potential
- Depletion Approximation
- Electrostatics Calculations & Diagrams

Assignments:
Reading: Streetman and Banerjee §5.2-5.3
Homework 3 due tomorrow, Friday Sep 26th by 5pm
Exam 1 this Tuesday Sept. 30th

p-n Junction

- Most semiconductor devices contain at least one junction between p-type and n-type material
- So what happens when we bring together a slab of n-type material and a slab of p-type material?
- We already have most of the tools to understand this
- An understanding of p-n junctions is needed to analyze the other devices we will look at in this class

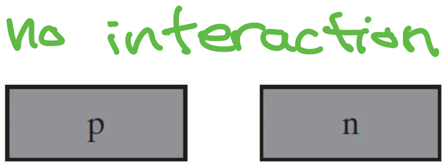


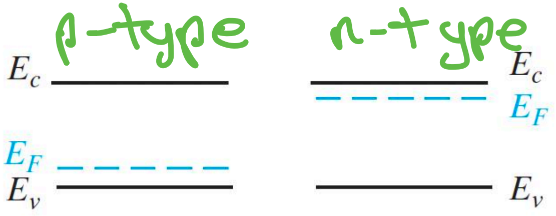
Assumptions for this lecture: 1D current flow, uniform cross-sectional area, in equilibrium (no external excitation)

Sources: Wikipedia

Equilibrium Conditions: Isolated Regions

- Recall: If you have isolated regions of n- and p-type material, we know what our band diagrams look like
- Equilibrium means no external excitation and no net current flow
- First, we will look at the step/abrupt *junction*
  - uniform p doping on one side of a sharp junction and uniform n doping on the other side
- Before joining, the n-type material has a large concentration of e^-
- The p-type material has a large concentration of h^+





(a)

Sources: Textbook

Equilibrium Conditions: Step/Abrupt Junction

- Immediately upon joining, there is a large concentration gradient of carriers across the sample
- Holes diffuse from the p side into the n side
- Electrons diffuse from n to p
- An opposing E-field is created at the junction and builds up until the net current is 0
  - Recall:

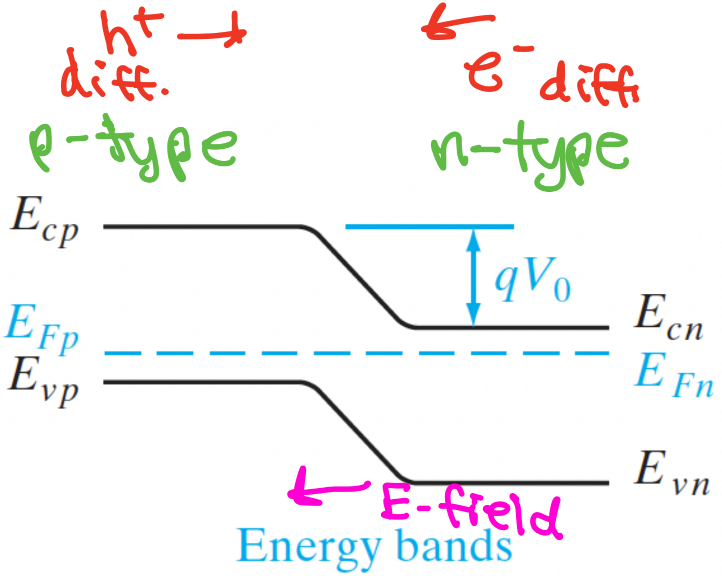
$$J_p(\text{drift}) + J_p(\text{diff.}) = 0$$

$$J_n(\text{drift}) + J_n(\text{diff.}) = 0$$

- What does the Fermi level look like in equilibrium?

  E_F is constant, no slope

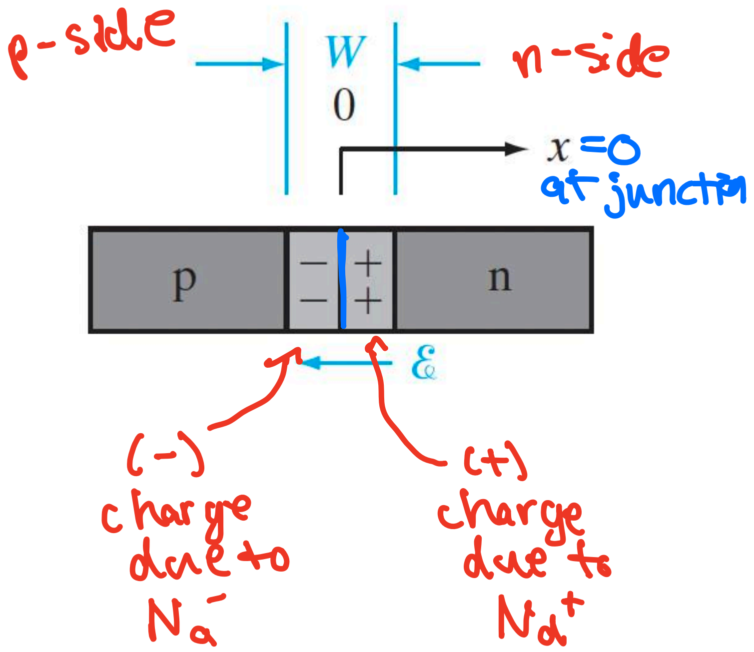
- We can now draw the band diagram



Sources: Textbook

Space Charge in the Transition Region

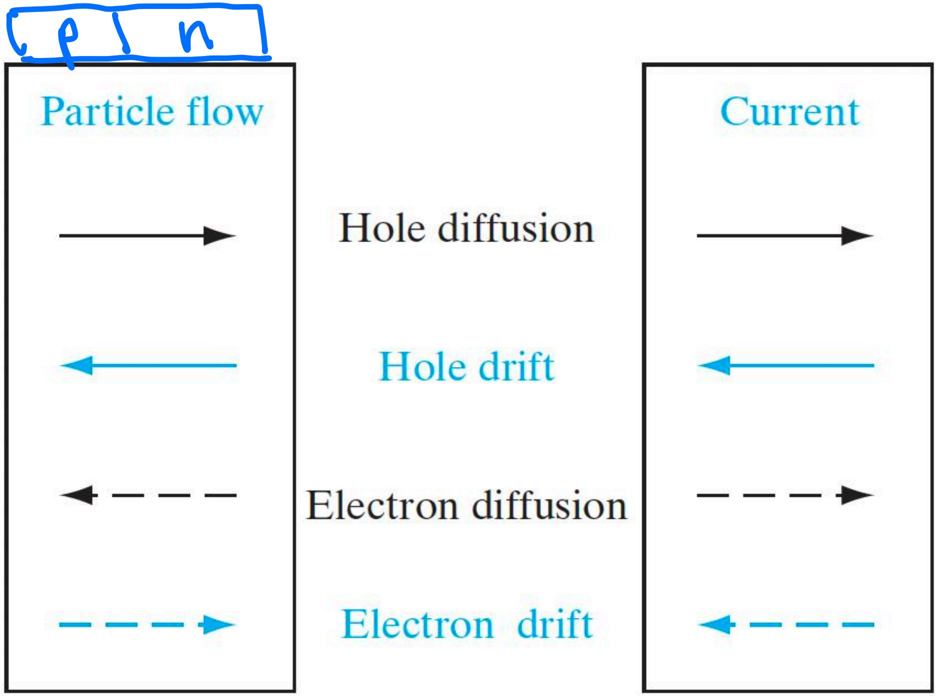
- e- diffusing from n to p side will leave behind N_d^+
- h+ diffusing from p to n side will leave behind N_a^-
- After diffusion is done,
- This leads to the development of a region of positive space charge near the n side of the junction and negative charge near the p side
- This region is denoted W, also called the depletion/transition
- The E-field appears across this region near the junction (W)



Sources: Textbook

Visualizing Directions of Particle and Current Flow

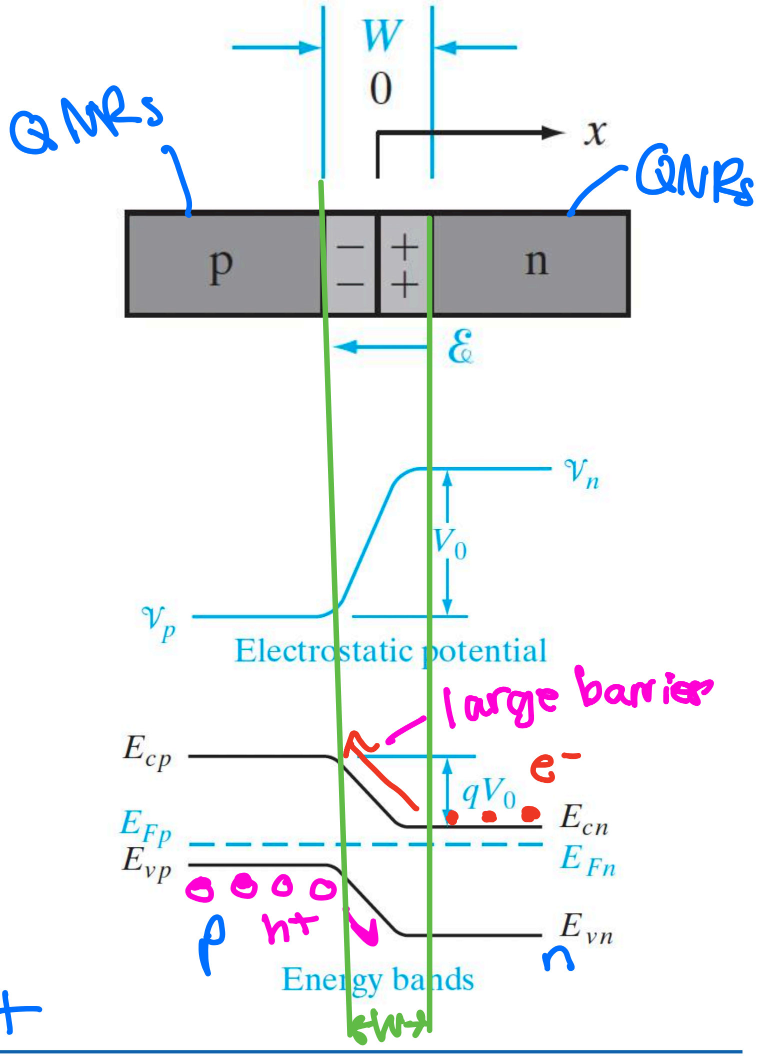
- Directions of the four components of particle flow within the transition region, and the resulting current directions
- Recall: electron current is opposite to the direction of flow of electrons



Sources: Textbook

Contact Potential

- There is an equilibrium potential difference $\underline{V_0}$ across W
- Gradient in potential oppose to the direction of E-field $\mathcal{E}(x) = -d(V)/dx$
- We assume E-field is $\underline{0}$ in the $\underline{\text{quasi-neutral}}$ regions outside W (QNRs)
- The contact potential, $\underline{V_0} = V_n - V_p$
  - V_0 is a built-in potential barrier
  - Necessary to maintain equilibrium at the junction
- Can you measure it with a voltmeter? No, external potential will preclude measurement



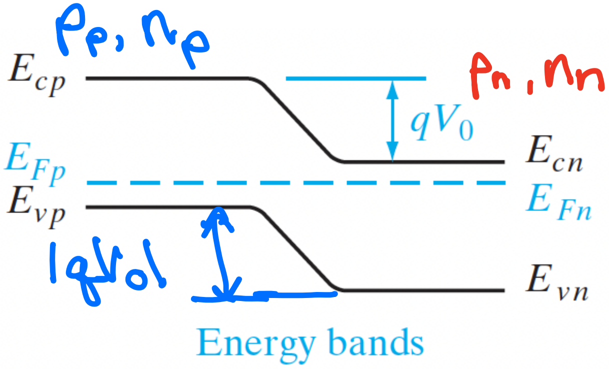
Sources: Textbook

Calculating Contact Potential

- From the band diagram,

$$qV_0 = E_{vp} - E_{vn} = |E_{cp} - E_{cn}|$$

$$qV_0 = (E_{ip} - E_F) + (E_F - E_{in})$$



$$E_{ip} - E_F = kT \ln \frac{p_p}{n_i}$$

$$E_F - E_{in} = kT \ln \frac{n_n}{n_i}$$

- We can write V_0 in terms of the the acceptor and donor concentrations on the p- and n-sides, respectively:

$$V_0 = \frac{kT}{q} \ln \frac{N_a}{n_i^2/N_d} = \boxed{\frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}}$$

- Equilibrium conditions still hold true:

$$\text{p-side} \quad p_p n_p = n_i^2 = p_n n_n \quad \text{n-side}$$

- What do the subscripts in n and p denote? which side of the junction

Sources: Textbook

Carriers in a p-n Junction

full ionized

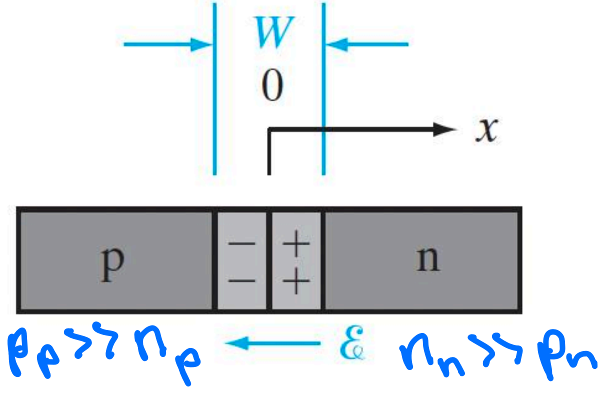
- Notation:
  - On the p-side majority carrier: $p_p \approx$ <u>Na</u> outside of W
  - On the n-side, majority carrier $n_n \approx$ <u>Nd</u> outside of W

- Using $n_p p_p = n_i^2$ on p-side, minority carriers there
  - $n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_a}$; $p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_d}$

- From the built-in voltage: $$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$$

- This relates the majority/minority carrier concentration on either side of the junction

- Which becomes more useful next lecture(s) when we apply an <u>external voltage</u>



Sources: E. Pop E340 Slides, Textbook

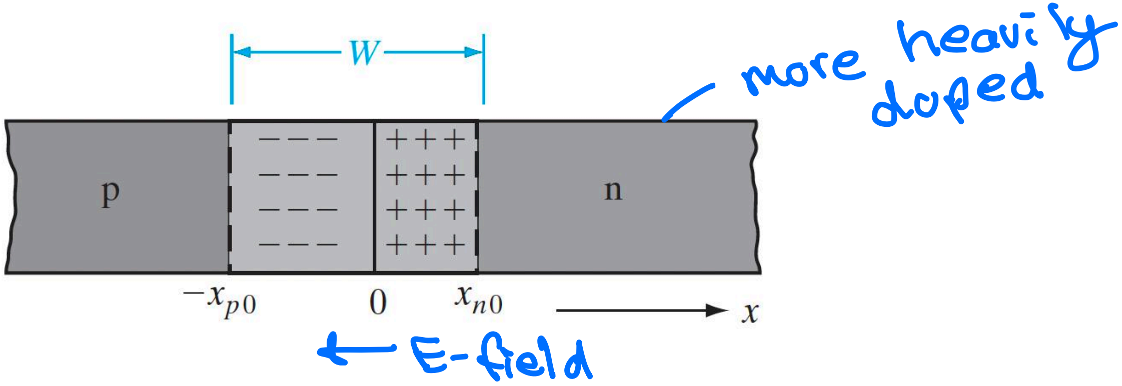
Depletion Approximation

- We approximate there is an abrupt junction between the space charge ($N_D - N_A$) region and the two quasi-neutral (n and p) regions

- e- and h+ are in transit in W from one side of the junction to the other. Some electrons diffuse from n to p
- The E-field serves to sweep out carriers which have wandered into W
- Results: There are very few carriers in the transition region *W* at any time

- We consider the space charge within W is due only to N_a^- and N_d^+

Space charge and E-field distribution within the transition region of a p-n junction with $N_d > N_a$:



Sources: Textbook

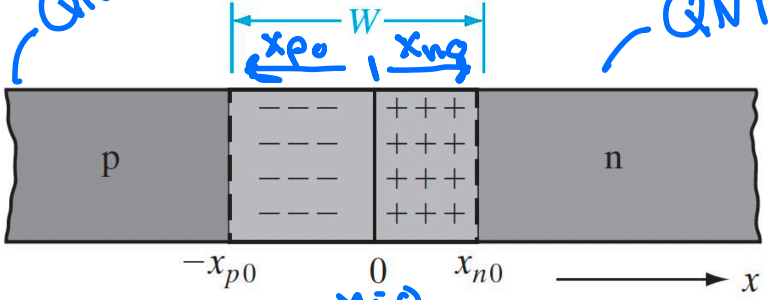
Depletion Approximation Continued

- What is the depletion region? Where e^- and h^+ are gone
- What is the space charge region (SCR)? Same thing as depletion region
- What are the quasi-neutral regions (QNR)? p and n regions that have free carriers

- If the SCR width is $W = x_{p_0} + x_{n_0}$, do the two (x_{p_0}, x_{n_0}) sides have to be equal? Why or why not? No, depends on # of dopants!

QNR SCR QNR

Space charge and E-field distribution within the transition region of a p-n junction with $N_d > N_a$: $x_{n0} < x_{p0}$



x=0 at junction

Sources: E. Pop E340 Slides, Textbook

p-n Junction Electrostatics: Charge

Given a sample with cross-sectional area *A*,

- Charge on the p-side:

$$Q_- = Q_p = -qAx_{p0}N_a$$

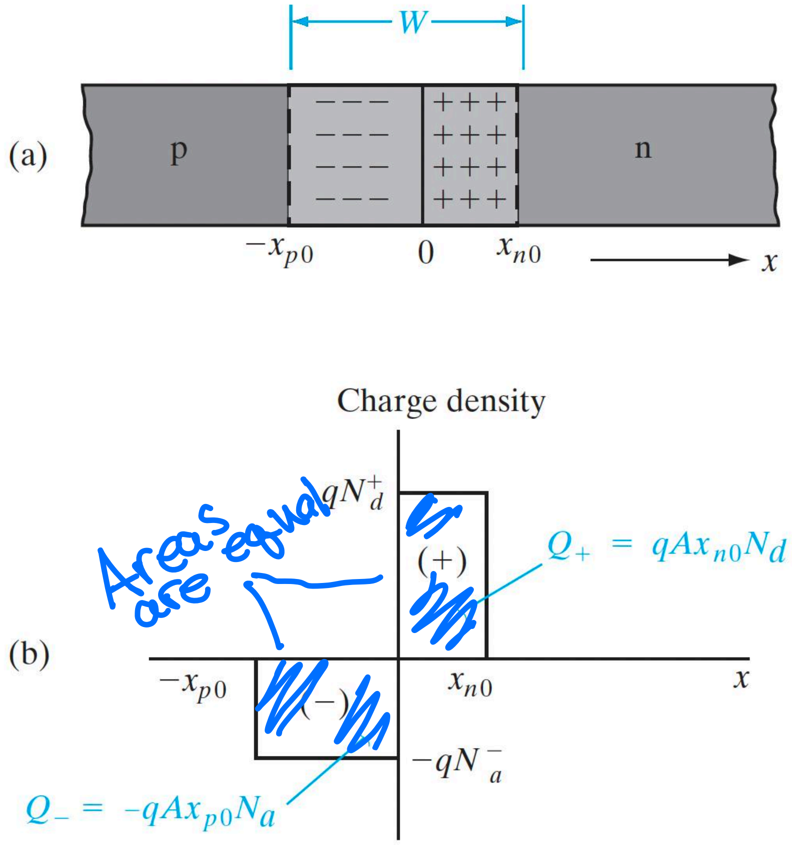
- Charge on the n-side:

$$Q_+ = Q_n = +qAx_{n0}N_a$$

- Total charge on either side of the junction:

$$|Q_p| = |Q_n|$$

$$qAx_{p0}N_a = qAx_{n0}N_d$$



Sources: E. Pop E340 Slides, Textbook

p-n Junction Electrostatics: E-Field

- To calculate the E-field within W, we can use Poisson's equation, which relates the gradient of the E-field to the local space charge at any point x:

no/very few carriers in SCR

$$\frac{d\mathscr{E}(x)}{dx} = \frac{q}{\epsilon}(\cancel{p} - \cancel{n} + N_d^+ - N_a^-)$$

- If we neglect p and n in the SCR, and assuming complete ionization, we can simplify

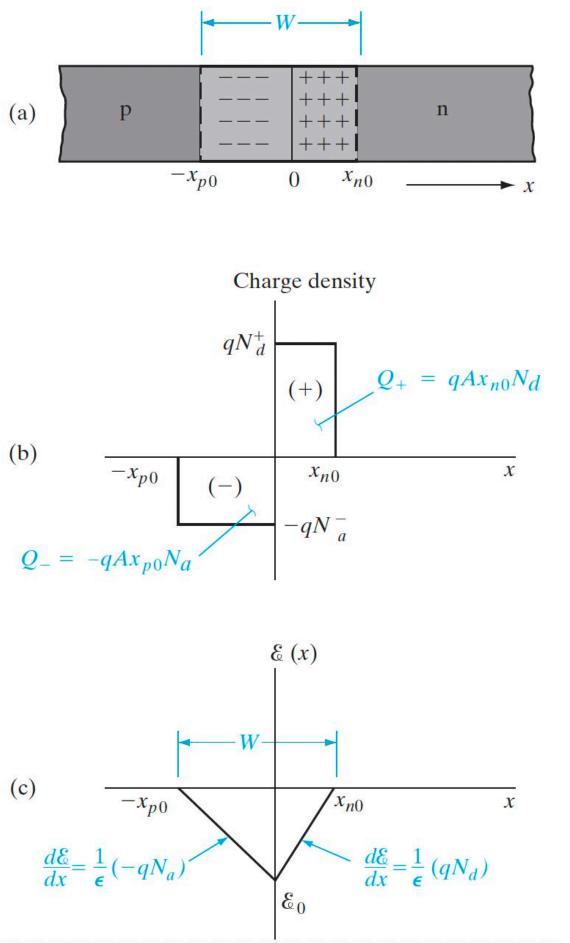
$$\frac{d\mathscr{E}}{dx} = \frac{q}{\epsilon}N_d, \qquad 0 < x < x_{n0}$$

n-region of SCR

permittivity = $\epsilon_0 \epsilon_r$

$$\frac{d\mathscr{E}}{dx} = -\frac{q}{\epsilon}N_a, \qquad -x_{p0} < x < 0$$

p-region of SCR



Sources: Textbook

p-n Junction Electrostatics: E-Field Continued

- We can easily relate E-field to __V₀__ (the field at any x is the negative of the potential gradient at that point)

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx} \quad \text{or} \quad -V_0 = \int_{-x_{p0}}^{x_{n0}} \mathscr{E}(x)dx$$

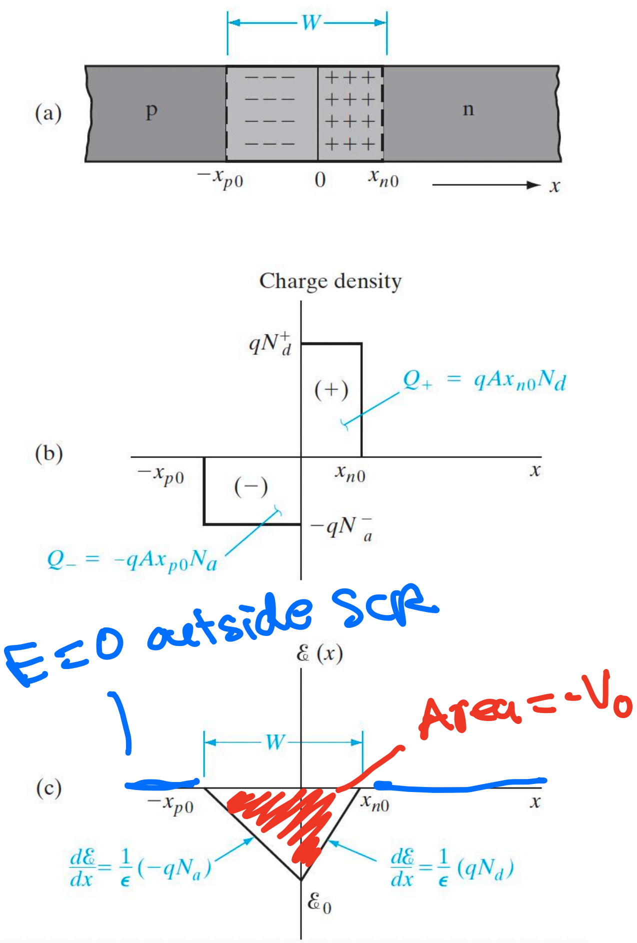
- Therefore, the negative of the contact potential is simply the __area__ under the E(x) versus x triangle!
  - This relates the contact potential to the width of the depletion region:

$$V_0 = -\frac{1}{2}\mathscr{E}_0 W = \frac{1}{2}\frac{q}{\epsilon}N_d x_{n0} W$$

  - Which can also be written,

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}\frac{N_a N_d}{N_a + N_d}W^2$$

$\mathcal{E}_0 = E_0 \rightarrow$ peak E-field at $x=0$



Sources: Textbook

p-n Junction Electrostatics: E-Field

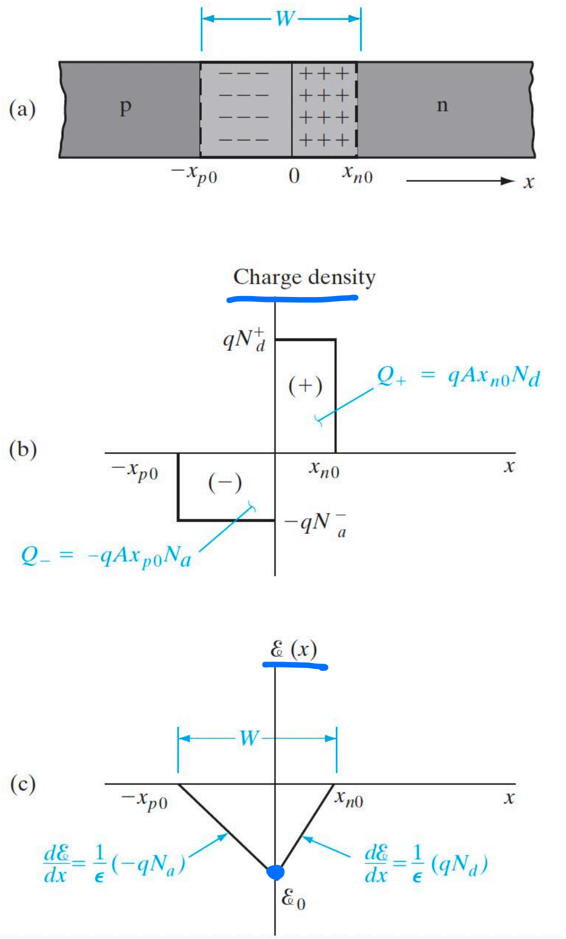
- What does the E-field look like? Triangular
- Positive slope (E-field increasing with x) on the n-side
- Negative slope (E-field becomes more negative as x increases) on the p-side
- Max value E_0 at x = 0

$$\int_{\mathscr{E}_0}^{0} d\mathscr{E} = \frac{q}{\epsilon}N_d\int_0^{x_{n0}} dx, \qquad 0 < x < x_{n0}$$

$$\int_0^{\mathscr{E}_0} d\mathscr{E} = -\frac{q}{\epsilon}N_a\int_{-x_{p0}}^{0} dx, \quad -x_{p0} < x < 0$$

$$\mathscr{E}_0 = -\frac{q}{\epsilon}N_d x_{n0} = -\frac{q}{\epsilon}N_a x_{p0}$$

- We know E-field points in –x direction (from n to p), so it is negative
- E-field assumed to go to zero outside of W!



Sources: Textbook

p-n Junction Electrostatics: Depletion Widths

- What about the depletion width, W?
- Rearranging V_0 equation,

$$\boxed{W = \left[\frac{2\epsilon V_0}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}} \qquad W = \left[\frac{2\epsilon kT}{q^2}\left(\ln\frac{N_a N_d}{n_i^2}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

- Note: transition/depletion width W varies as the $\sqrt{V}$ in this case, $\sqrt{V_0}$
- We can also calculate the penetration of the transition region into the n and p materials:

$$x_{p0} = \frac{WN_d}{N_a + N_d} = \frac{W}{1 + N_a/N_d} = \left\{\frac{2\epsilon V_0}{q}\left[\frac{N_d}{N_a(N_a + N_d)}\right]\right\}^{1/2}$$

$$x_{n0} = \frac{WN_a}{N_a + N_d} = \frac{W}{1 + N_d/N_a} = \left\{\frac{2\epsilon V_0}{q}\left[\frac{N_a}{N_d(N_a + N_d)}\right]\right\}^{1/2}$$

- As we expect, this predicts transition region extends further into side with lightly! (i.e. if $N_a \ll N_d$, x_{p0} is large compared with x_{n0})

Sources: Textbook

Problem: p-n Junctions in Equilibrium

- An abrupt Si p-n junction has $N_a = 10^{18}\ \mathrm{cm}^{-3}$ on one side and $N_d = 5 \times 10^{15}\ \mathrm{cm}^{-3}$ on the other. Calculate the Fermi level positions at 300 K in the p and n regions. Draw an equilibrium band diagram for the junction and determine the contact potential V_0 from the diagram. Compare the results with V_0 calculation.

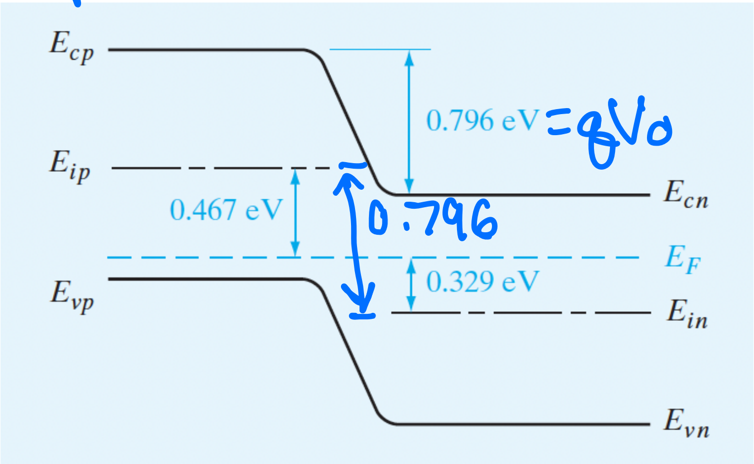
on p-side!

$$E_{ip} - E_F = kT\ln\left(\frac{p_p}{n_i}\right) = 0.026\ln\left(\frac{10^{18}}{1.5\times10^{10}}\right) = 0.467\ eV$$

$$E_F - E_{in} = kT\ln\left(\frac{n_n}{n_i}\right) = 0.026\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right) = 0.329\ eV$$

$$qV_0 = (E_{ip} - E_F) + (E_F - E_{in}) = 0.467 + 0.329 = 0.796\ eV$$

p-side n-side



- Alternatively,

$$qV_0 = kT\ln\left(\frac{N_aN_d}{n_i^2}\right) = 0.026\ln\left(\frac{(10^{18})(5\times10^{15})}{(1.5\times10^{10})^2}\right)$$
$$= 0.796\ eV$$

Sources: Textbook

Problem: p-n Junctions in Equilibrium

- Consider the same sample (an abrupt Si p-n junction has $N_a = 10^{18}\ cm^{-3}$ on one side and $N_d = 5 \times 10^{15}\ cm^{-3}$ on the other) with a circular cross section of $10\ \mu m$. Calculate x_{n0}, x_{p0}, Q_+, and E_0 for this junction at equilibrium (300 K). Sketch E(x) and charge density to scale.

$$W = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{\frac{1}{2}} = \left[\frac{(2)(11.8)(8.85 \times 10^{-14})(0.796)}{1.6 \times 10^{-19}}\left(\frac{1}{10^{18}}+\frac{1}{5 \times 10^{15}}\right)\right]^{\frac{1}{2}} = 0.457\ \mu m$$

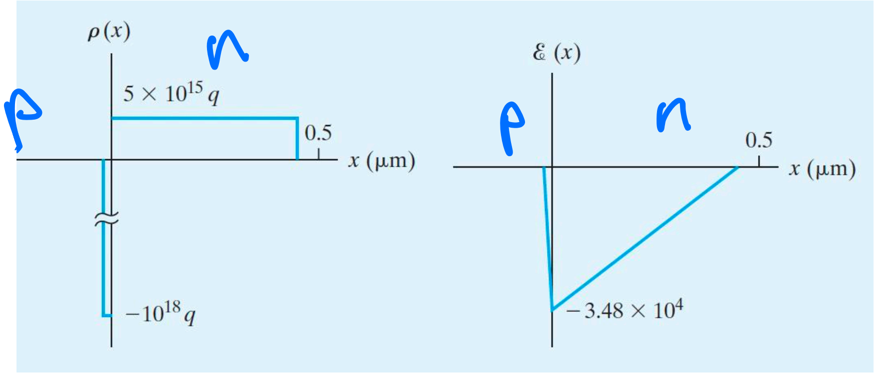
$$x_{n_0} = \frac{W}{1+\frac{N_d}{N_a}} = \frac{0.457}{1+5 \times 10^{-3}} = 0.455\ \mu m$$

$$x_{p_0} = \frac{W}{1+\frac{N_a}{N_d}} = \frac{0.457}{1+200} = 2.27 \times 10^{-3}\ \mu m$$

$$A = \pi r^2 = \pi(5 \times 10^{-4}\ cm)^2 = 7.85 \times 10^{-7} cm$$

$$Q_+ = qAx_{n_0}N_d = (1.6 \times 10^{-19})(7.85 \times 10^{-7})(0.455 \times 10^{-4})(5 \times 10^{15}) = qAx_{p_0}N_a = 2.85 \times 10^{-14}\ C$$

$$E_0 = -\frac{qx_{n_0}N_d}{\epsilon} = -\frac{q|x_{p_0}|N_a}{\epsilon} = \frac{-(1.6 \times 10^{-19})(0.455 \times 10^{-4})(5 \times 10^{15})}{(11.8)(8.85 \times 10^{-14})} = 3.48 \times 10^{4}\ V/cm$$



Sources: Textbook